5/7/18 "Wherever smart people work, doors are unlocked."-Steve Wozniak

HW: "Frequency and Period of Sinusoidal Graphs" Homework section

AIM: What are the Frequency and Period of Sinusoidal graphs?

Warm Up:

Name: ____________________ Date: ______________

Basic Graphs of Sine and Cosine
Common Core Algebra II Homework

Fluency

1. On the grid below, sketch the graphs of each of the following equations based on the basic sine function.

$y = \sin(x)$

$y = 3\sin(x)$

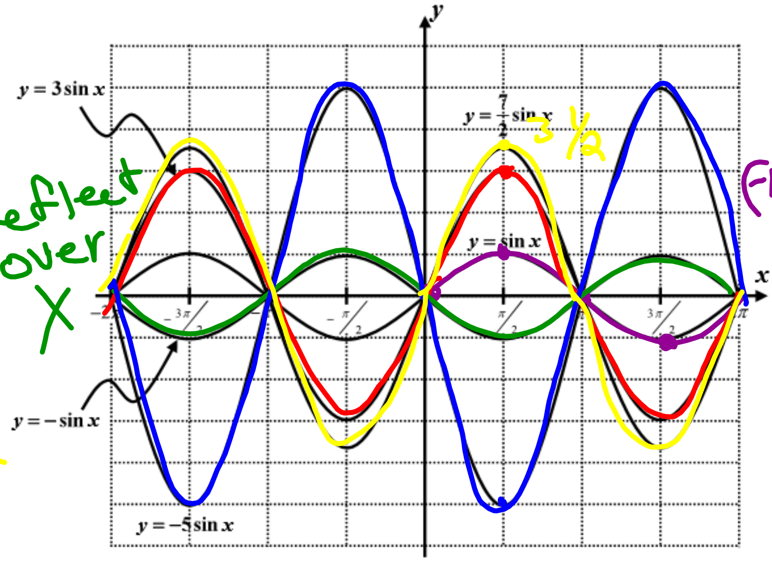
$y = -\sin(x)$

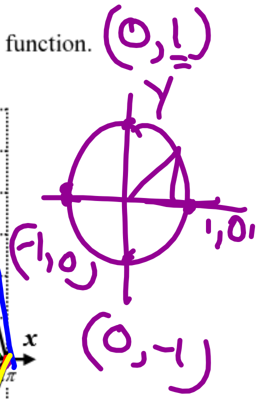
$y = -5\sin(x)$

$y = \frac{7}{2}\sin(x)$

reflect over x

3.5





2. On the grid below, sketch the graphs of each of the following equations based on the basic cosine function.

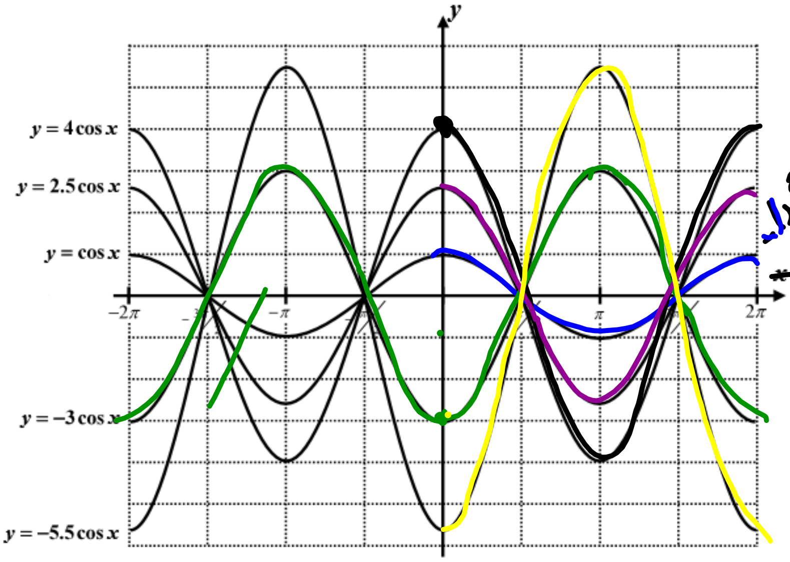
$y = \cos(x)$

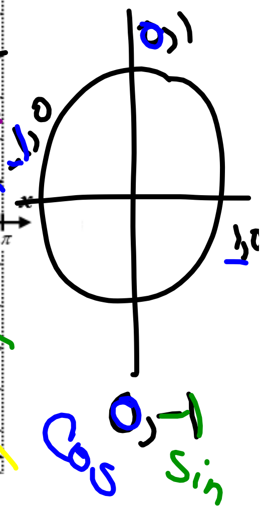
$y = 4\cos(x)$

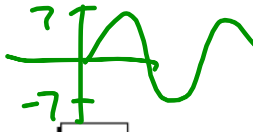
$y = -3\cos(x)$

$y = 2.5\cos(x)$

$y = -5.5\cos(x)$







3. Which of the following represents the *range* of the trigonometric function $y = 7\sin(x)$?

(1) $(-7, 7)$ (3) $[0, 7)$

(2) $[-7, 7]$ (4) $(-7, 7]$

Since the amplitude is now 7, the sine graph must rise and fall 7 units above the x-axis.

(2)

4. Which of the following is the period of $y = \cos(x)$?

(1) π (3) 2π

(2) 2 (4) $\frac{3\pi}{2}$

The basic cosine and sine graphs will repeat themselves every 2π units along the x-axis.

(3)

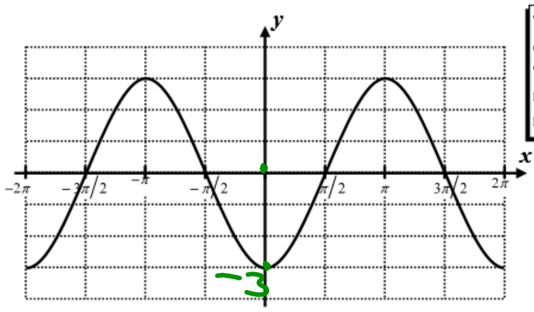
5. Which of the following equations describes the graph shown below?

(1) $y = 3\cos(x)$

(2) $y = -3\cos(x)$

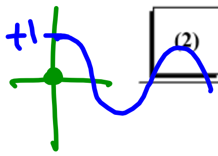
(3) $y = 3\sin(x)$

(4) $y = -3\sin(x)$



This graph is clearly based on cosine, but because it "starts" at a minimum, it must be multiplied by a negative.

cos



(2)

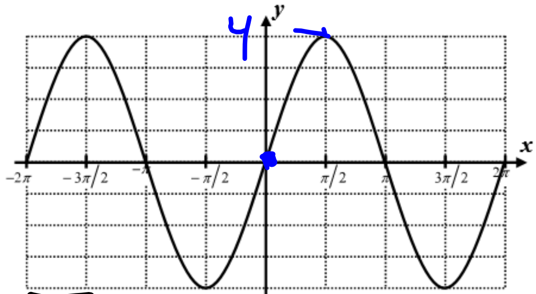
6. Which of the following equations represents the periodic curve shown below?

(1) $y = 4\cos(x)$

(2) $y = -4\cos(x)$

(3) $y = 4\sin(x)$

(4) $y = -4\sin(x)$



This is the basic sine graph that has been stretched by a factor of 4 in the vertical direction. Since it "starts" by increasing out of the origin, it is multiplied by a positive.

(3)

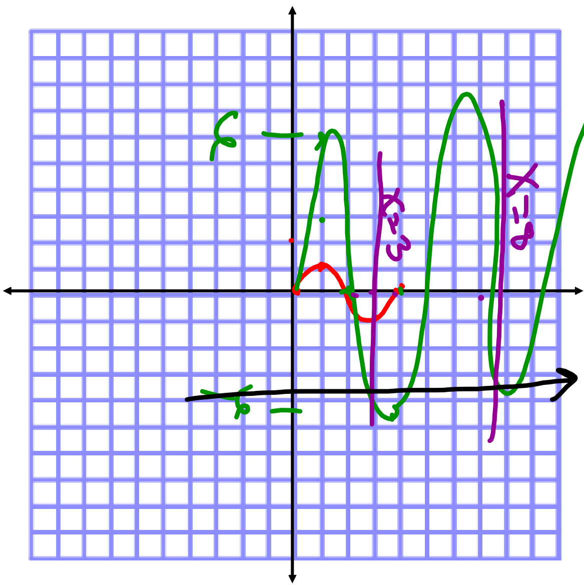
7. Which of the following lines when drawn would *not* intersect the graph of $y = 6\sin(x)$?

(1) $x = 8$ (3) $y = -4$

(2) $x = 3$ (4) $y = 9$

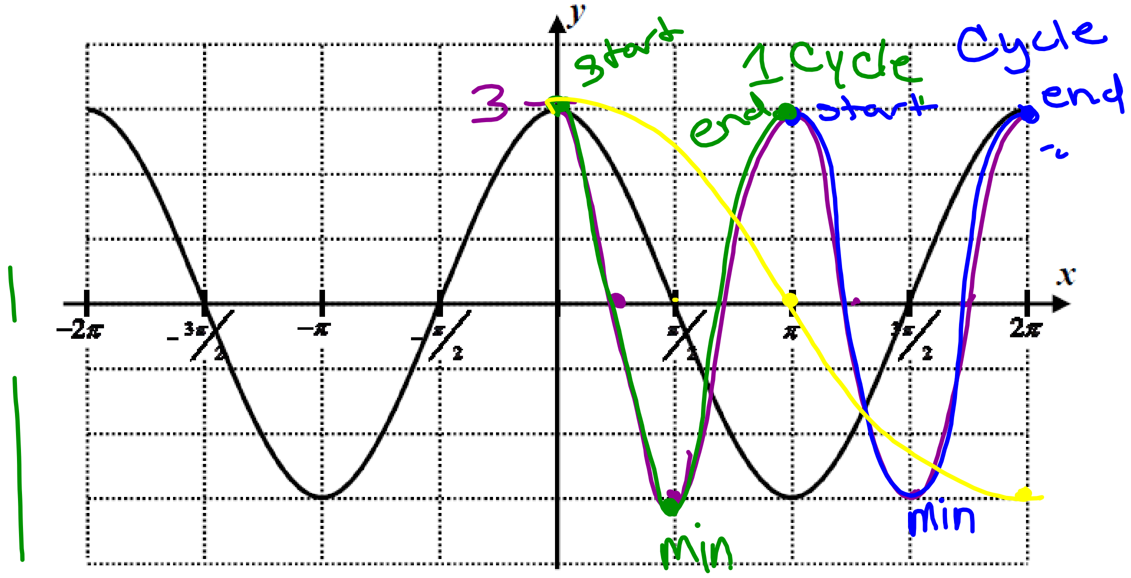
The range of this sine function is $[-6, 6]$; thus, the graph will never hit the horizontal line $y = 9$.

(4)



Y = Sin X

Exercise **#1:** On the grid below is a graph of the function $y = 3\cos(x)$.



(a) Using your calculator, sketch the graph of $y = 3\cos(2x)$ on the same axes.

The graph Compressed by 2.

(b) How many full cycles or periods of this function now fit within 2π radians?

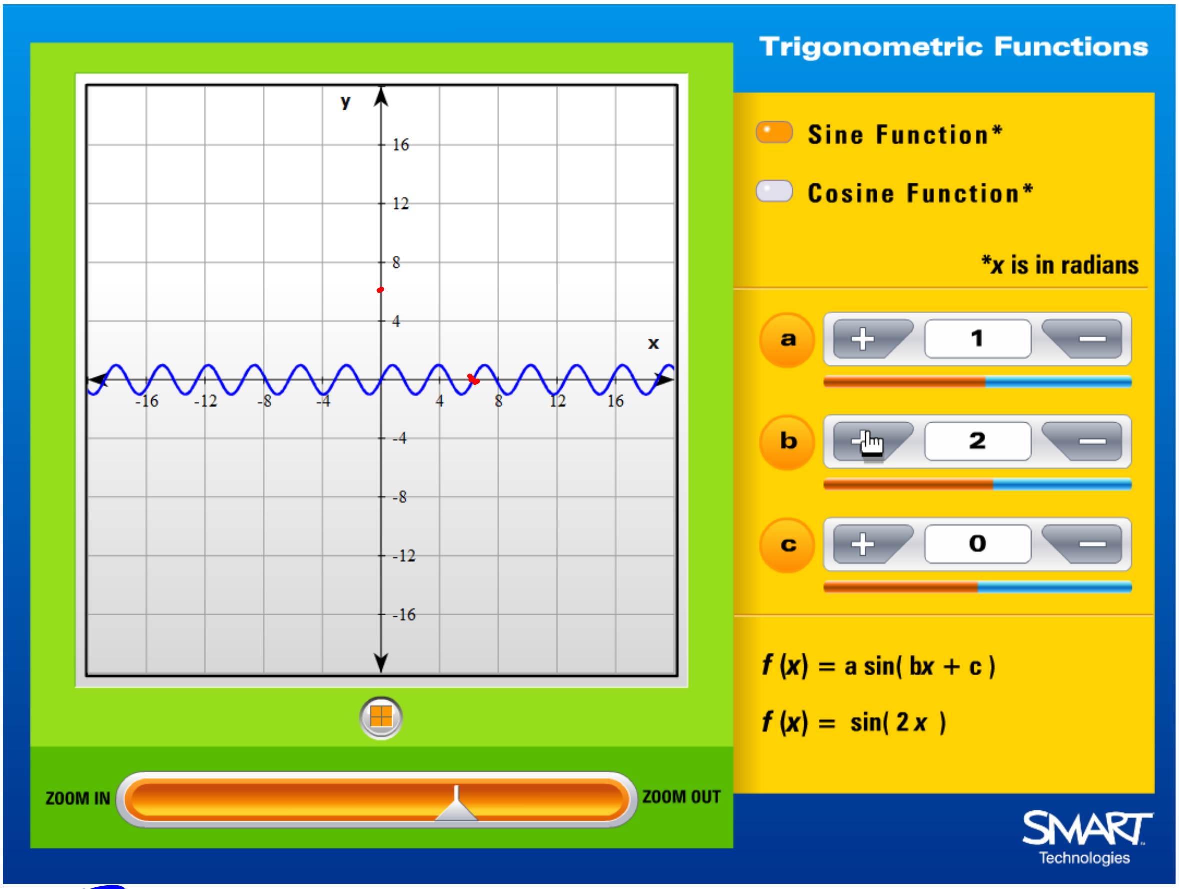
2 Cycles

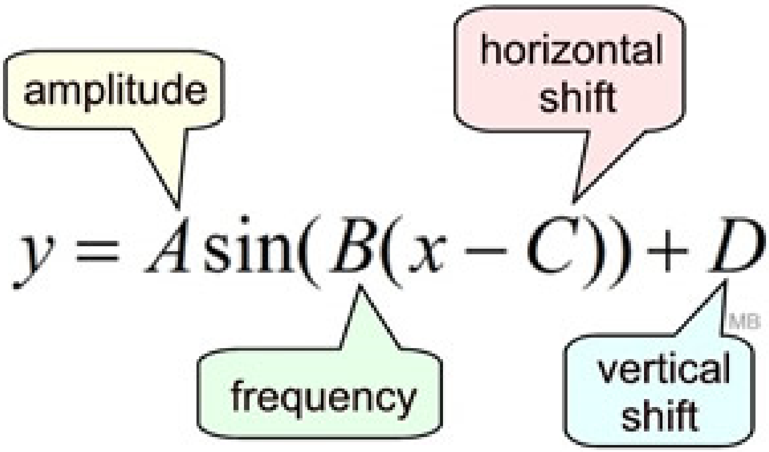
(c) Using your calculator, sketch the graph of $y = 3\cos\left(\frac{1}{2}x\right)$ on the same axes.

This Strectches the graph by 2

(d) How many full cycles or periods of this function now fit within 2π radians?

1/2 a Cycle


Trigonometric Functions
Sine Function*
Cosine Function*
*x is in radians
a
1
b
2
c
0
f (x) = a sin(bx + c)
f (x) = sin(2 x)
ZOOM IN
ZOOM OUT
y
x
16
12
8
4
-4
-8
-12
-16
SMART Technologies


amplitude
horizontal shift
$y = A\sin(B(x - C)) + D$
frequency
vertical shift

How much for one cycle

The **period**, P, of a sinusoidal function is an extremely important concept. It is defined as **the minimum horizontal shift needed for the function to repeat its fundamental pattern.** The period for the basic sinusoidal graphs is 2π. Clearly, from our first exercise, the period of the function depends on the coefficient B in the general equations $y = A\sin(Bx)$ and $y = A\cos(Bx)$. This coefficient, B, is known as the **frequency**.

amplitude — horizontal shift

$y = A\sin(B(x-C))+D$

frequency — vertical shift

$Y = A\sin(B(x-C))+D$

frequency

Exercise #2: Consider the graphs from *Exercise* #1. For each below, state the frequency and period.

(a) $y = 3\cos(x)$ 1

Frequency, $B = 1$

Period, $P = 2\pi$

(b) $y = 3\cos(2x)$

Frequency, $B = 2$

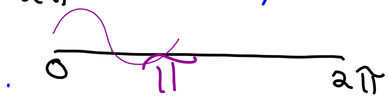
Period, $P = \pi$

(c) $y = 3\cos\left(\frac{1}{2}x\right)$

Frequency, $B = \frac{1}{2}$

Period, $P = 4\pi$

$\boxed{2\pi\cdot\frac{2}{1} = 4\pi}$



$P = \frac{2\pi}{\frac{1}{2}}$

Clearly we can see from *Exercise* #2 that the frequency and period are **inversely related**, that is as one increases the other decreases and vice versa.

Exercise #3: Examine the results from *Exercise* #2. What is true about the product of the period, P, and the frequency, B? Write an equation for this relationship.

multiply

$2\pi\cdot 1 = 2\pi$ $2\cdot\pi = 2\pi$ $\frac{1}{2}\cdot 4\pi = 2\pi$

$(\text{Frequency})(\text{Period}) = 2\pi$ $\text{Period} = \frac{2\pi}{\text{freq}}$

Exercise #4: Determine the period of each of the following sinusoidal functions. Express your answers in exact form.

Frequency is usually designated by "b"

(a) $y = 6\sin(4x)$

$b = 4$

$\frac{4(P)}{4} = \frac{2\pi}{4}$

$P = \frac{2\pi}{4} = \frac{1\pi}{2} = \frac{\pi}{2}$

$\text{Period} = \frac{\pi}{2}$

(b) $y = 8\cos\left(\frac{\pi}{3}x\right)$

$\left(\frac{3}{\pi}\right)\frac{\pi}{3}(\text{Period}) = 2\pi\left(\frac{3}{\pi}\right)$

$\boxed{\text{Period} = 6}$

(c) $y = -12\sin\left(\frac{2}{3}x\right)$

$\left(\frac{3}{2}\right)\frac{2}{3}(\text{Period}) = 2\pi\left(\frac{3}{2}\right)$

$\boxed{\text{Period} = 3\pi}$

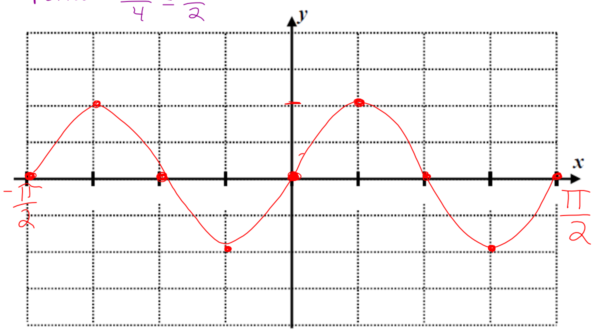
Exercise #5: Sketch the function $y = 2\sin(4x)$ on the grid below for one full period to the left and right of the y-axis. Label the scale on your axes.

amplitude 2 frequency

sin

$4(\text{Period}) = 2\pi$

$\text{Period} = \frac{2\pi}{4} = \frac{\pi}{2}$



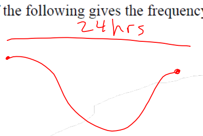
Exercise #6: The heights of the tides can be described using a sinusoidal model of the form $y = A\cos(Bx)+C$. If high tides are separated by 24 hours, which of the following gives the frequency, B, of the curve?

(1) 12

(2) $\frac{\pi}{24}$

(3) $\frac{\pi}{12}$

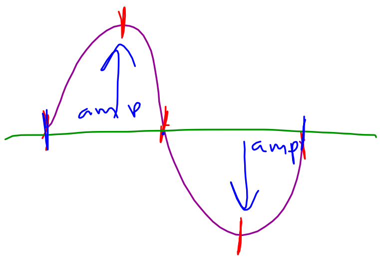
(4) $\frac{\pi}{6}$



$24\text{hrs} = \text{Period}$

$\frac{24(\text{freq})}{24} = \frac{2\pi}{24} = \frac{2\pi}{24} = \frac{\pi}{12}$

sin



cos